Risk Attribution in a Portfolio

BY NATHAN FABER / ON JULY 28, 2014

Diversification is touted as the only free lunch (see our old post Is Diversification Really a Free Lunch) in investing and is a primary way to reduce portfolio volatility without sacrificing a proportional amount of return. Return characteristics aside, a well-diversified portfolio can be less risky than any of the constituents taken alone; it is truly a case where the sum of the parts is greater than the whole.

It is this complex interaction among the individual assets that makes risk attribution interesting when examining portfolios. One asset can look very different when it is taken in the context of two different portfolios.

Return attribution is a simple, linear calculation. When we calculate the return of a portfolio, we can simply take the weighted average of the individual asset returns. That is, for n assets, the portfolio return, R, is given by:

$$R = \sum_{i}^{n} w_i r_i$$

where r_i is the return on asset i and w_i is the weight of asset i in the portfolio.

For portfolio risk attribution, one formula often seen utilizes the marginal contribution to risk (MCTR):

With this formula, we can use estimates of the covariance and overall portfolio variance to calculate the MCTR. Additionally, we can substitute back into the first equation for volatility and rearrange to get:

$$\sigma = \sum_{i}^{n} w_i \sigma_i \rho(r_i, R)$$

where $\rho(r_i, R)$ is the correlation between the i^{th} asset return and the overall portfolio return. This looks more like our additive return attribution equation although it is still nonlinear due to the dependence of R on w.

Intuitively, the assets with higher weight, higher volatility, and a greater alignment with the portfolio return will contribute the most to the portfolio risk.

As an example, we will consider adding an asset, Emerging Market Equities (EEM), to an existing 60/40 SPY/TLT portfolio. Using 2013 data, the risk contribution from SPY and TLT for different allocations is shown in the following graph.

$$\sigma = \sum_{i}^{n} w_i MCTR_i$$

which appears to be a nice, linear equation, as well, until we look into how we calculate MCTR:

$$MCTR_i = \frac{\partial \sigma}{\partial w_i}$$

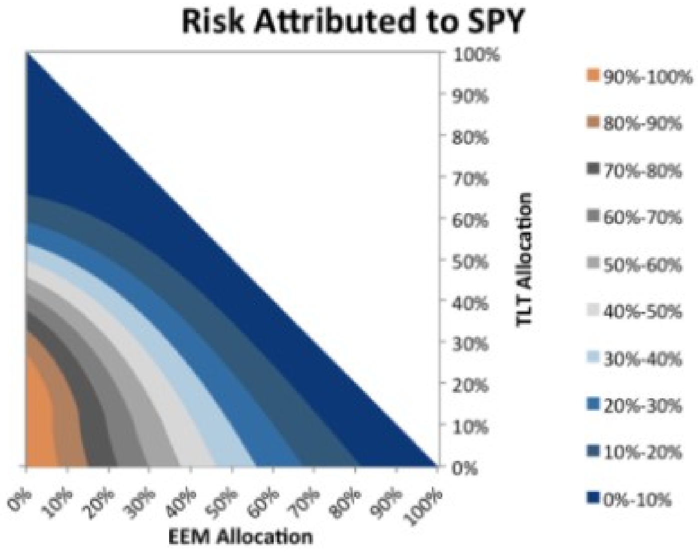
The dependence of σ on a derivative of σ introduces nonlinearity when the assets are correlated. "Marginal" refers to the incremental risk introduced in the portfolio for a given change in asset allocation. Estimating derivatives such as MCTR can be tough when smoothness is not guaranteed. However, with a little mathematical manipulation, we work this into a more intuitive form. The volatility can be written using the asset covariances:

$$\sigma = \sqrt{\sum_{i}^{n} \sum_{j}^{n} w_i w_j cov(r_i, r_j)}$$

This is more complicated than the weighted average of the standard deviations, and it is only a weighted average of the variances (weighted by the square of the asset weights) when all of the assets are uncorrelated.

Differentiating this formula with respect to w_i yields:

$$MCTR_i = \frac{\partial \sigma}{\partial w_i} = \frac{1}{\sigma} \sum_{j}^{n} w_j cov(r_i, r_j)$$


Risk Attributed to SPY
TLT Allocation
EEM Allocation
0%
10%
20%
30%
40%
50%
60%
70%
80%
90%
100%
90%-100%
80%-90%
70%-80%
60%-70%
50%-60%
40%-50%
30%-40%
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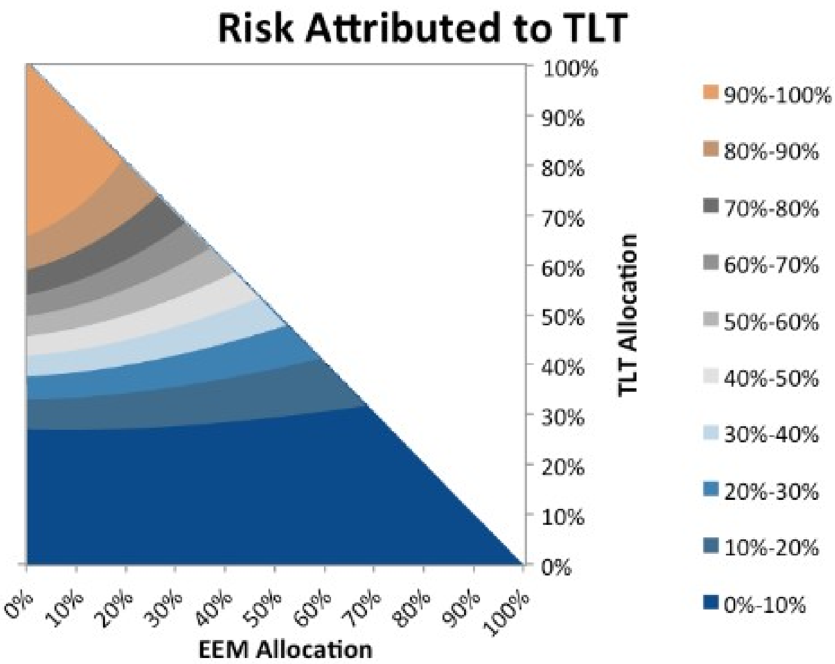
Risk Attributed to TLT
TLT Allocation
EEM Allocation
0%
10%
20%
30%
40%
50%
60%
70%
80%
90%
100%
90%-100%
80%-90%
70%-80%
60%-70%
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10%-20%
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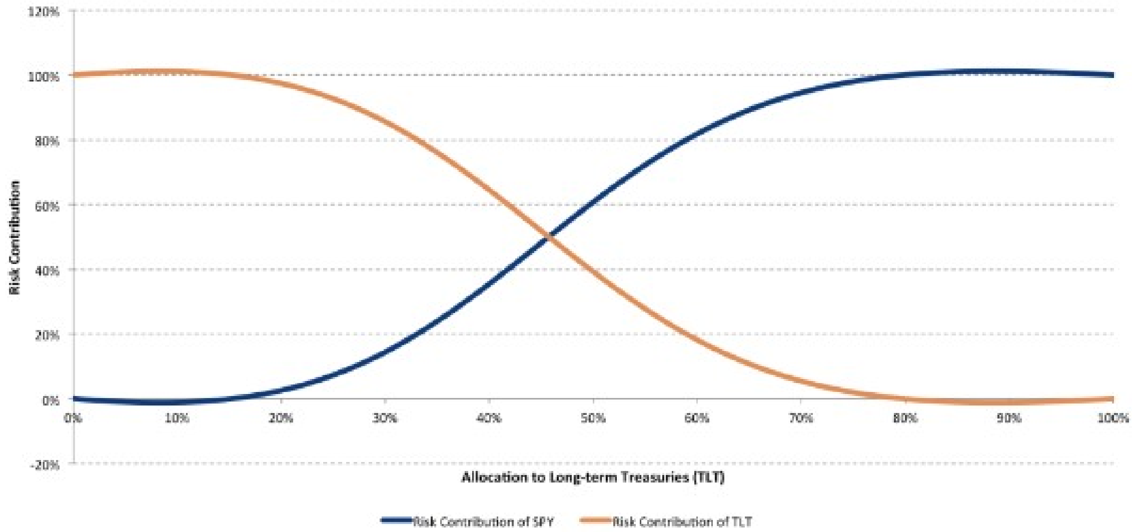
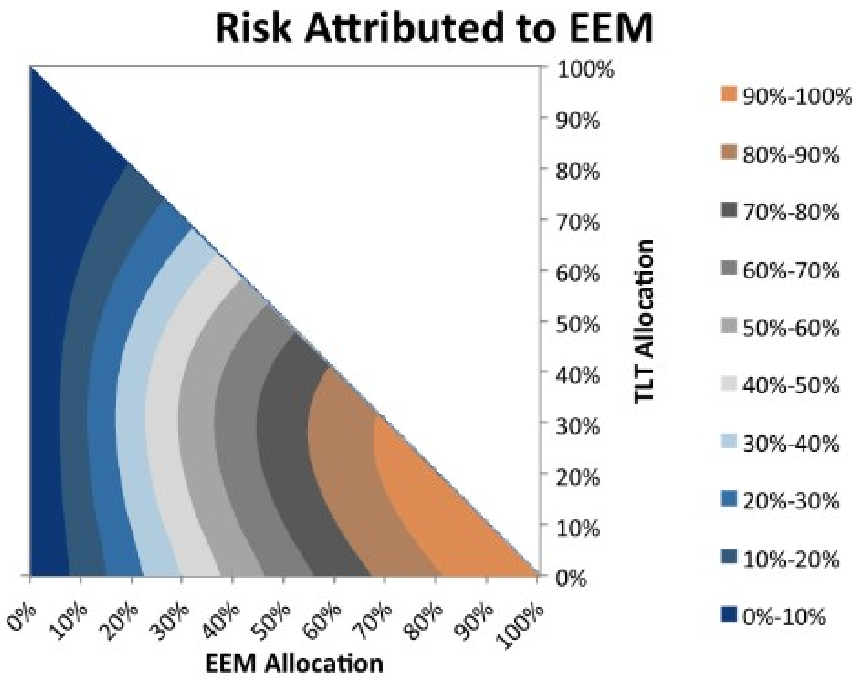
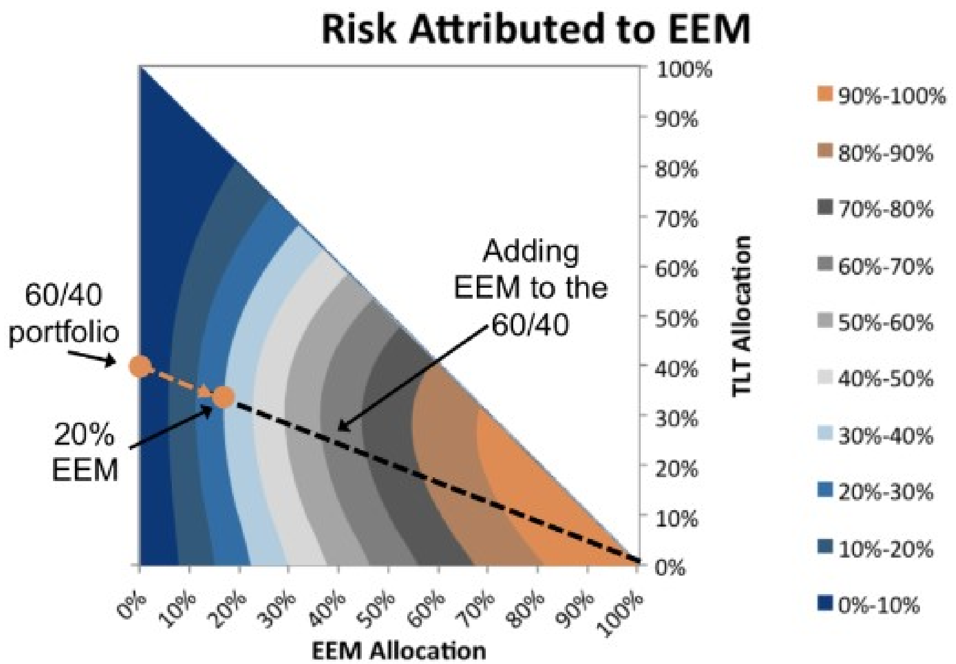


The 60/40 portfolio had about 40% of its volatility from SPY and 60% from TLT over this time period. When EEM is added to the portfolio, we have a 3D surface of risk contribution.

The three plots below show how the risk contribution of each asset varied for different portfolio compositions.



For this example, we want to see how adding EEM to our 60/40 affects the risk profile. From the following graph, we can see that an EEM allocation of 20% would have contributed about 30% to the overall portfolio risk profile.



These graphical methods are not practical in cases where we have more than three assets. However, the formulas and the intuition gained for how different assets contribute to the risk do carry over. Too often investors focus on what assets are driving returns without carefully assessing the incremental risk attributable to each one. The qualitative labels of "low risk" and "high risk" are not applicable across the board – context is required.

Nathan Faber

Nathan is a Vice President at Newfound Research, a quantitative asset manager offering a suite of separately managed accounts and mutual funds. At Newfound, Nathan is responsible for investment research, strategy development, and supporting the portfolio management team.

Prior to joining Newfound, he was a chemical engineer at URS, a global engineering firm in the oil, natural gas, and biofuels industry where he was responsible for process simulation development, project economic analysis, and the creation of in-house software.

Nathan holds a Master of Science in Computational Finance from Carnegie Mellon University and graduated summa cum laude from Case Western Reserve University with a Bachelor of Science in Chemical Engineering and a minor in Mathematics.